HEAT EXCHANGE BETWEEN ROTATING DISKS

V. V. Nikulin

UDC 536.24.01

The generalization of [1] in this paper is similar to the generalization made in [2] with respect to [3] without the energy equation taken into account. Investigation of the ordinary differential equations derived in the linear approximation for the case of heat transfer of a rotating disk into an unbounded rotating fluid has shown that the ratio of the thickness of the thermal boundary layer to the thickness of the hydrodynamical one for a fixed Prandtl number depends only on the ratio of the angular velocities of the fluid and the disk and tends to infinity when they are equal. Thus rotating systems provide an example of the kind of motion in which different physical mechanisms are responsible for the formation of the thermal and hydrodynamical boundary layers. Thus the thermal layer is produced as a result of the fact that the flow of fluid from infinity to the disk prevents unlimited heat diffusion, i.e., a limit to diffusion occurs due to convection. The hydrodynamical layer is a layer of the Ekman type [4] and is produced on account of a balance of Coriolis and drag forces.

1. A half-space filled with a viscous incompressible fluid and bounded by an infinite disk is discussed. The fluid at an infinite distance from the disk has a temperature T_∞ and is rotating with angular velocity Ω. The disk has a temperature T_0 and is rotating with angular velocity $\alpha\Omega$.

The continuity, Navier–Stokes, and energy equations in a cylindrical coordinate system with axis coinciding with the rotation axis and with the axial symmetry of the flow taken into account (we assume a fluid density $\rho = 1$) have the form

$$\frac{1}{r}\frac{\partial (ur)}{\partial r} + \frac{\partial w}{\partial z} = 0,$$

$$\begin{gathered}
u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2}\right),\\
u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right),\\
u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \nu\left(\frac{\partial^2 v}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) - \frac{v}{r^2}\right),\\
u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \varkappa\left(\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right) +\\
+ \frac{2\nu}{c_V}\left(e_{rr}^2 + e_{\varphi\varphi}^2 + e_{zz}^2 + 2e_{r\varphi}^2 + 2e_{\varphi z}^2 + 2e_{rz}^2\right),
\end{gathered} \tag{1.1}$$

where ν is the kinematic viscosity, $\varkappa$ is the thermal conductivity coefficient, c_V is the specific heat of the fluid at constant volume, and e_{ij} is the deformation rate tensor; ν, $\varkappa$, and c_V are assumed to be constant, and (uwv) are the radial, axial, and rotational components of the velocity in the cylindrical coordinate system $(r\,z\,\varphi)$.

Following [1, 2], we will seek a solution of (1.1) in the form

$$\begin{gathered}
v/r = \Omega g(\zeta),\ w = (\nu\Omega)^{1/2}h(\zeta),\ u/r = -(\Omega/2)dh/d\zeta,\\
p = p(z) + (1/2)\Omega^2 r^2,\\
T = (\nu\Omega/c_V)(\xi^2 S(\zeta) + q(\zeta)) + T_\infty,\\
z = (\nu/\Omega)^{1/2}\zeta,\ r = (\nu/\Omega)^{1/2}\xi.
\end{gathered} \tag{1.2}$$

Then the system (1.1) decomposes into two equations, the first of which is solved independently of the second:

$$\begin{cases} \frac{1}{4}h'^2 - \frac{1}{2}hh'' - g^2 = -1 - \frac{1}{2}h''', \\ -gh' + g'h = g''; \end{cases} \tag{1.3}$$

$$\begin{cases} S'' - \sigma hS' + \sigma h'S = -\sigma\left(g'^2 + \frac{1}{4}h''^2\right), \\ q'' - \sigma hq' = -(4S + 3\sigma h'^2) \end{cases} \tag{1.4}$$

with the boundary conditions

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 119–124, January–February, 1979. Original article submitted January 24, 1978.

$$\begin{cases} h(0) = h'(0) = 0, \ g(0) = \alpha & \text{at} \quad \zeta = 0, \\ h' \to 0, \qquad g \to 1 & \text{as} \quad \zeta \to \infty; \end{cases} \tag{1.5}$$

$$\begin{cases} S(0) = 0, \ q(0) = \frac{c_V}{\nu\Omega}(T_0 - T_\infty) & \text{at} \ \zeta = 0, \\ S \to 0, \quad q \to 0 & \text{as} \ \zeta \to \infty \end{cases} \tag{1.6}$$

($\sigma = \nu/\varkappa$ is the Prandtl number).

The authors of [2, 5-7] investigated Eqs. (1.3) with the conditions (1.5). If now the solution of the system (1.3) is known, it is possible to investigate Eqs. (1.4) according to [1]. We note that the heat transfer from a rotating disk into a nonrotating fluid has been generalized in [8] to the case of compressibility and a linear temperature dependence of the viscosity. Nonsteady heat transfer in the case in which the temperature of the disk was varied instantaneously has been discussed in [9].

Let us consider the case in which the rotational velocity of the disk differs little from the rotational velocity of the fluid at infinite distance from the disk, i.e., $|1-\alpha| \ll 1$. Then as follows from [4, 6], Eqs. (1.3) are linearized, and their solution, which satisfies the boundary conditions (1.5), has the form

$$g(\zeta) = 1 + (\alpha - 1)\cos(\zeta)\exp(-\zeta),$$
$$h(\zeta) = -(\alpha - 1)(1 - \sin(\zeta)\exp(-\zeta) - \cos(\zeta)\exp(-\zeta)).$$

It follows from this that the thickness of the hydrodynamical boundary layer does not depend on α and is a quantity ~ 1 with respect to the variable ζ, i.e., $\sim(\nu/\Omega)^{1/2}$ in real coordinates.

Let us estimate the size of the thermal boundary layer. Using the results of [1], the solution of the first equation from (1.4) satisfying (1.6) can be written in the form

$$S(\zeta) = y_1 \left\{ \frac{\left(\int\limits_{\zeta}^{\infty} \frac{du}{y_1^2}\right)\left[\int\limits_0^{\infty} \frac{1}{y_1^2}\left(\int\limits_0^t M(u)\, y_1(u)\, du\right) dt\right]}{\int\limits_0^{\infty} \frac{du}{y_1^2}} - \int\limits_{\zeta}^{\infty} \frac{1}{y_1^2}\left(\int\limits_0^t M(u)\, y_1(u)\, du\right) dt \right\} \exp[-\beta(\zeta + \cos(\zeta)\exp(-\zeta))], \tag{1.7}$$

where

$$\beta = \frac{\sigma(\alpha-1)}{2}; \quad M(\zeta) = -\sigma\left(g'' + \frac{1}{4}h'''\right)\exp\left(-\frac{\sigma}{2}\int h\, d\zeta\right) =$$
$$= -2\sigma(\alpha-1)^2 \exp[-2\zeta + \beta(\zeta + \cos(\zeta)\exp(-\zeta))],$$

and $y_1(\zeta)$ is some fundamental solution of the homogeneous equation

$$y_1'' + \left(\frac{3}{2}\sigma h' - \frac{\sigma^2 h^2}{4}\right) y_1 = 0, \tag{1.8}$$

which is obtained from the first equation of (1.4) by the substitution $y = S(\zeta)\exp\left(-\frac{\sigma}{2}\int h\, d\zeta\right)$; furthermore, y_1 satisfies the condition

$$\int\limits_0^{\infty} \frac{d\zeta}{y_1^2} < \infty. \tag{1.9}$$

Having substituted expressions for h' and h into (1.8), we obtain

$$y_1'' + [-\beta^2 - 2\beta(3-\beta)\sin(\zeta)\exp(-\zeta) - \beta^2\exp(-2\zeta)(1 + \sin(2\zeta)) + 2\beta^2\cos(\zeta)\exp(-\zeta)]\, y_1 = 0. \tag{1.10}$$

We will find the asymptote of $y_1(\zeta)$ from (1.10) for the investigation of the function $S(\zeta)$ at large ζ.

1. Let $\alpha > 1$, i.e., $\beta > 0$. Let us denote $t = \beta\zeta$ and $dy_1/dt = x$; then we obtain from (1.10) the system

$$dy_1/dt = x, \quad dx/dt = \varphi(t) y_1, \tag{1.11}$$

where

$$\varphi(t) = 1 + \exp\left(-\frac{2t}{\beta}\right)\left(1 + \sin\left(\frac{2t}{\beta}\right)\right) + \frac{2}{\beta}(3-\beta)\sin\left(\frac{t}{\beta}\right)\exp\left(-\frac{t}{\beta}\right) - 2\cos\left(\frac{t}{\beta}\right)\exp\left(-\frac{t}{\beta}\right).$$

It is possible to asymptotically integrate the system (1.11) by means of reducing it to the L-diagonal form [10]. Asymptotic integration is possible in the region $t_0 \le t < \infty$, in which $\varphi(t) \ge k > 0$ is valid. Assuming $\beta \le 1/10$ and setting $t_0 = 2\beta \ln(1/\beta)$, we obtain that the inequality $\varphi(t) \ge k > 0$ is satisfied for $t \in [t_0, \infty)$.

Let us take as the fundamental solution of (1.11) on $[t_0\infty)$

$$y_1=(\gamma+\eta)\exp\left(\int\limits_{t_0}^{t}\sqrt{\varphi}\,du\right),\quad x=(\gamma-\eta)\exp\left(\int\limits_{t_0}^{t}\sqrt{\varphi}\,du\right),\tag{1.12}$$

where γ and η satisfy the system of integral equations

$$\begin{aligned}\gamma&=1+\int\limits_{t}^{\infty}\frac{\varphi'}{4\varphi}(\gamma-\eta)\,du,\\ \eta&=\int\limits_{t_0}^{t}\frac{\varphi'}{4\varphi}(\gamma-\eta)\exp\left(-2\int\limits_{u}^{t}\sqrt{\varphi}\,du\right)du.\end{aligned}\tag{1.13}$$

One can check that the y_1 and x so determined satisfy the system (1.11). As has been shown in [10], Eqs. (1.13) have a unique continuous bounded solution on $[t_0\infty)$. Then we obtain from (1.13)

$$\gamma=1+O\left(\frac{1}{\beta}\exp\left(-\frac{t}{\beta}\right)\right),\quad \eta=O(\beta\exp(-2t)).$$

We note that $\gamma\to 1$ and $\eta\to 0$ for any t from $[t_0\infty)$ as $\beta\to 0$. And thus with account taken of the fact that $\sqrt{\varphi}=1+f(t)$, where $|f(t)|<(7/\beta)\exp(-t/\beta)$, we obtain from (1.12) an estimate $y_1(t)$ for sufficiently small β and $t\in[t_0\infty)$.

Now we estimate $y_1(t)$ on the interval $[0t_0)$. It follows from the boundedness of γ and η on $[t_0\infty)$ for $\beta\leq 1/10$ and from (1.12) that $y_1(t_0)$ and $y'_1(t_0)$ are also bounded and $y_1(t_0)>0$, $y'_1(t_0)>0$ for sufficiently small β. Then we have for $y_1(t)$

$$y_1(t)=y_1(t_0)+y_1'(t_0)(t-t_0)+\int\limits_{t}^{t_0}du\int\limits_{u}^{t_0}\varphi(u)\,y_1(u)\,du.\tag{1.14}$$

Let us denote

$$\delta=\int\limits_{0}^{t_0}du\int\limits_{0}^{t_0}|\varphi(u)|\,du.$$

For sufficiently small β, $\delta<1$ is satisfied; moreover, $\delta\to 0$ as $\beta\to 0$. Then we obtain from (1.14)

$$\sup_{t\in[0t_0]}|y_1(t)-y_1(t_0)|\leqslant\frac{|y_1(t_0)|\,\delta+|y_1'(t_0)|\,t_0}{1-\delta}.$$

Thus for sufficiently small β the quantity $y_1(t)>0$ and is bounded on $[0t_0]$, since $y_1(t_0)\to 1$ as $\beta\to 0$. Due to this and the asymptotic Eq. (1.12), condition (1.9) is satisfied.

Using (1.12) and the expression for $\varphi(t)$, we obtain from (1.7) the asymptote of $S(\zeta)$ for $\zeta>\zeta_0$, where $\zeta_0=t_0/\beta=2\ln(1/\beta)$,

$$S(\zeta)=O(\beta^2\exp(-2\beta\zeta)).\tag{1.15}$$

From this it follows that the thickness of the thermal boundary layer is $\sim 1/2\beta$, or $\sim(\nu/\Omega)^{1/2}(1/2\beta)$ in real con- coordinates, i.e., it exceeds the thickness of the hydrodynamical layer by a large factor for small β.

2. Let $\alpha<1$, i.e., $\beta<0$. Denoting $t=-\beta\zeta$, we obtain a system similar to (1.11), only β is replaced by $-\beta$ in the expression for $\varphi(t)$. Carrying out the same procedure that we did in the case $\alpha>1$, we find the asymptote $y_1(\zeta)$. It will be expressed similarly to (1.12), i.e., $y_1(\zeta)\sim\exp(-\beta\zeta)$. Substituting the latter into (1.7), we obtain that $S(\zeta)$ does not tend to zero as $\zeta\to\infty$. Thus in the case in which the fluid is rotating more rapidly at infinity than the disk, there is no stationary solution. Physically, this is explained by the fact that under these conditions the fluid flows away from the disk and heat diffusion is in no way restricted.

The expression for q is found by direct integration of the second equation of (1.4)

$$q=\frac{c_V(T_0-T_\infty)}{\nu\Omega\,J_1}\int\limits_{\zeta}^{\infty}\exp\left(\sigma\int hdu\right)du+\frac{J_2}{J_1}\int\limits_{\zeta}^{\infty}\exp\left(\sigma\int hdu\right)du-\int\limits_{\zeta}^{\infty}\exp\left(\sigma\int hdu\right)\left[\int\limits_{0}^{u}N(u)\exp\left(-\sigma\int hdu\right)du\right]du,\tag{1.16}$$

where

$$J_1=\int\limits_{0}^{\infty}\exp\left(\sigma\int hdu\right)du;$$

$$J_2=\int\limits_{0}^{\infty}\left[\exp\left(\sigma\int hdu\right)\int\limits_{0}^{u}N(u)\exp\left(-\sigma\int hdu\right)du\right]du;$$

$$N(\zeta) = -(4S + 3\sigma h'^2) = -4\left(S + \frac{12}{\sigma}\beta^2 \sin^2(\zeta)\exp(-2\zeta)\right).$$

We obtain

$$q = O(\exp(-2\beta\zeta))$$

from (1.15) and (1.16). It is also evident from (1.16) that there exists a solution for q satisfying the boundary conditions (1.6) only for $\beta > 0$, even if one neglects dissipation, i.e., sets $N(\zeta) = 0$.

2. The flow between two coaxial infinite rotating disks situated at a distance l from each other is discussed. The temperatures and angular velocities of the disks are T_1, Ω_1 and T_2, Ω_2. The flow between infinite coaxial parallel disks has been investigated in [2, 5, 11]. The problem can be reduced, just as in the case of a single disk, to a system of ordinary differential equations. The difference consists of the fact that the pressure is sought in the form $p = P(z) + (1/2)\lambda^2 r^2$ instead of (1.2), where λ is found from the boundary conditions. Thus it has been shown in [5] that if the rotational velocities of the disks are similar $\lambda = (\Omega_1 + \Omega_2)/2$. The boundary conditions (1.6) are replaced by

$$S(0) = S(l) = q(l) = 0,\quad q(0) = (c_V/\nu\Omega)(T_1 - T_2).$$

If h and g are known, the functions S and q are determined from the relationships

$$S(\zeta) = y_1\left[\frac{-\left(\int\limits_0^\zeta \frac{du}{y_1^2}\right)\left(\int\limits_0^l \frac{1}{y_1^2}\left(\int\limits_0^u M(u)\,y_1(u)\,du\right)du\right)}{\int\limits_0^l \frac{du}{y_1^2}} + \int\limits_0^\zeta \frac{1}{y_1^2}\left(\int\limits_0^u M(u)\,y_1(u)\,du\right)du\right]\exp\left(\frac{\sigma}{2}\int h\,d\zeta\right),$$

where y_1 is some fundamental solution of the homogeneous Eq. (1.8);

$$q(\zeta) = \frac{c_V}{\nu\Omega}(T_1 - T_2)\left[1 - \frac{\int\limits_0^\zeta \exp\left(\sigma\int h\,du\right)du}{I_1}\right] -$$

$$-\frac{I_2}{I_1}\int\limits_0^\zeta \exp\left(\sigma\int h\,du\right)du + \int\limits_0^\zeta \exp\left(\sigma\int h\,du\right)\left(\int\limits_0^u N(u)\exp\left(-\sigma\int h\,du\right)du\right)du,$$

where

$$I_1 = \int\limits_0^l \exp\left(\sigma\int h\,du\right)du;$$

$$I_2 = \int\limits_0^l \exp\left(\sigma\int h\,du\right)\int\limits_0^u N(u)\exp\left(-\sigma\int h\,du\right)du\,du.$$

After the temperature distribution is found, one can calculate the heat transfer from any part of the disk.

The author is grateful to B. A. Lugovtsov for supporting this research.

LITERATURE CITED

1. K. Millsaps and K. Pohlhausen, "Heat transfer by laminar flow from a rotating plate," J. Aeron. Sci., 19, 120 (1952).
2. G. K. Batchelor, "Note on a class of solutions of Navier–Stokes equation representing steady rotationally symmetric flow," Quart. J. Mech. Appl. Math., 4, 29–41 (1951).
3. T. Von Kármán, "Über laminare und turbulent Reibung," Z. Angew. Math. Mech., 1, 233 (1921).
4. G. K. Batchelor, Introduction to Fluid Dynamics, Cambridge Univ. Press (1967).
5. K. Stewartson, "On the flow between two rotating coaxial disks," Proc. Cambr. Phil. Soc., 49, No. 2 (1953).
6. M. H. Rogers and G. N. Lance, "The rotationally symmetric flow of a viscous fluid in the presence of an infinite rotating disk," J. Fluid Mech., 7, 617 (1960).
7. P. D. Weidman and L. G. Redekopp, "On the motion of a rotating fluid in the presence of an infinite rotating disk," Arch. Mech., 28, Nos. 5–6 (1976).
8. N. Riley, "The heat transfer from a rotating disk," Quart. J. Mech. Appl. Math., 17, 331 (1964).

9. R. D. Andrews and N. Riley, "Unsteady heat transfer from a rotating disk," Quart. J. Mech. Appl. Math., 22, 19 (1969).
10. B. P. Demidovich, Lectures on Mathematical Stability Theory [in Russian], Nauka, Moscow (1967).
11. M. Holodnik, M. Kubicek, and V. Hlavacek, "Computation of the flow between two rotating coaxial disks," J. Fluid Mech., 81, Part 4, 689 (1977).

FILM CONDENSATION OF A MOVING VAPOR ON A HORIZONTAL CYLINDER

I. I. Gogonin, A. R. Dorokhov, and V. I. Sosunov

UDC 536.24:536.423

The computation of heat transfer during film condensation of a moving vapor on a horizontal cylinder is possible at this time for a whole series of simplifying hypotheses. The problem of an experimental confirmation of the dependences proposed [1-3] in order to refine further the mechanism of moving vapor condensation is urgent. There is a limited quantity of data in the literature which could be used for this purpose [4-8]. Their significant discrepancy under comparable conditions as well as the lack of systematic measurements on such important dependences for analysis as the dependence of the coefficient of heat transmission on the temperature head, the condensation pressure, and the geometric parameters should be noted.

This paper is a continuation of investigations on the condensation of the moving vapor Freon-21 (F-21, $CHFCl_2$) on horizontal cylinders. The tests were conducted on a test stand by the methodology of [8].

The tests were conducted on horizontally arranged nickel tubes of D = 16 mm outer diameter and L = 580 mm length, placed in a condenser with 400-mm inner diameter. The wall temperature t_W of the experimental sections was measured by six thermocouples calked around the perimeter of the tube at the middle of the section, and whose readings were averaged. The saturated vapor temperature t'' was measured by a thermocouple and was determined by means of the $\bar{p}-T$ data for Freon-21 by measuring the saturated vapor pressure with a standard manometer of the class 0.35.

The heat flux q on the outer surface of the experimental section was determined by the change in enthalpy of the cooling water which came in from a constant-level tank. The change in temperature head $\Delta t = t'' - t_W$ was achieved by adding hot water to the cooling water.

The ranges of variation of the main condensation parameters were $q = (3-150) \cdot 10^3$ W/m², $\Delta t = 1-30$°C, $t'' =$ 60-90°C. The accuracy of determining the heat transmission coefficient $\alpha = q/\Delta t$ at $\Delta t \geq 2$°C is estimated at 10%.

In the tests on moving vapor condensation the experimental sections were located in a channel whose geometry could be changed. The schemes for locating the experimental sections which were realized in the experiment are shown in Fig. 1a-c. In the case illustrated in Fig. 1a, the experimental section 2 is 170 mm from the vapor input to the channel. The spacing between the channel walls 1 was b = 26, 46, and 66 mm in the different test series. The vapor was smoothly introduced into the channel and three damper grids 4 were also set up. The spacing between the channel walls was 66 mm for the disposition of the experimental sections according to the scheme shown in Fig. 1b, and the tests were conducted serially in the 1, 4 and 9 tubes of a ten-set unstaggered bundle. In the case shown in Fig. 1c, the spacing between the walls was 26 mm and the inserts 3 simulating a checkerboard bundle with $s_1/D = 1.87$, $s_2/D = 0.81$ were additionally mounted in the channel. The tests were conducted in each tube of the bundle without feeding cooling water to those located above.

The experiment showed that despite the great diversity in conditions under which the test was conducted, the magnitude of the heat transmission coefficient had the same value upon referring the vapor velocity to the channel through-section. Part of the test results is presented in Fig. 2 for the vapor motion velocities w = 0.57 m/sec (points 1-3) and w = 1.1 m/sec (points 4 and 5) at $t'' = 60$°C. Points 1 and 4 correspond to the section arrangement shown in Fig. 1a, 2 to Fig. 1b, and 3, 5 to Fig. 1c. Tests conducted on different tubes of the bundle along its height at the same vapor velocity and saturation temperature showed that the coefficient of heat transmission has the identical value (within the limits of experimental error).

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 125-130, January-February, 1979. Original article submitted March 17, 1978.